Chapter 12

You're Getting Warm: Thermodynamics

In This Chapter

- Converting between temperature scales
- Working with linear expansion
- Calculating volume expansion
- Using heat capacities
- Understanding latent heat

Thermodynamics is the study of heat. It's what comes into play when you drop an ice cube into a cup of hot tea and wait to see what happens — if the ice cube or the tea wins out.

In physics, you often run across questions that involve thermodynamics in all sorts of situations. This chapter refreshes your understanding of the topic and lets you put it to use with practice problems that address thermodynamics from all angles.

Converting Between Temperature Scales

You start working with questions of heat by establishing a scale for measuring temperature. The temperature scales that you work with in physics are Fahrenheit, Celsius (formerly centigrade), and Kelvin.

Fahrenheit temperatures range from 32° for freezing water to 212° for boiling water. Celsius goes from 0° for freezing water to 100° for boiling water. Following are the equations you use to convert from Fahrenheit (F) temperatures to Celsius (C) and back again:

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}C + 32$$

The Kelvin (K) scale is a little different: Its 0° corresponds to *absolute zero,* the temperature at which all molecular motion stops. Absolute zero is at a temperature of –273.15° Celsius, which means that you can convert between Celsius and Kelvin this way:

$$K = C + 273.15$$

$$C = K - 273.15$$

To convert from Kelvin to Fahrenheit degrees, use this formula:

$$F = \frac{9}{5}(K - 273.15) + 32 = \frac{9}{5}K - 459.67$$

Technically, you don't say "degrees Kelvin" but rather "Kelvins," as in 53 Kelvins. However, people persist in using "degrees Kelvin," so you may see that usage in this book as well.

Q. What is 54° Fahrenheit in Celsius?

A. The correct answer is 12° C.

1. Use this equation:

$$C = \frac{5}{9}(F - 32)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32) = (0.55) \cdot (54 - 32) = 12°C$$

1. What is 23° Fahrenheit in Celsius?

Solve It

2. What is 89° Fahrenheit in Celsius?

Solve It

3. What is 18° Celsius in Fahrenheit?

Solve It

4. What is 18° Celsius in Kelvin?

Solve It

13. You're heating a 1.0 m^3 glass block, coefficient of volume expansion 1.0×10^{-5} °C^{-1}, raising its temperature by 27°C. What is the final volume of the block?

Solve It

14. You're heating a 3.0 m^3 gold block, coefficient of volume expansion 4.2×10^{-5} °C^{-1}, raising its temperature by 18°C. What is the final volume of the block?

Solve It

Getting Specific with Heat Capacity

It's a fact of physics that it takes 4186 J to raise the temperature of 1.0 kg of water by 1° C. But it takes only 840 J to raise the temperature of 1.0 kg of glass by 1° C.

You can relate the amount of heat, Q, it takes to raise the temperature of an object to the change in temperature and the amount of mass involved. Use this equation:

$$Q = m \cdot c \, \Delta T$$

In this equation, Q is the amount of heat energy involved (measured in Joules if you're using the MKS system), m is the amount of mass, ΔT is the change in temperature, and c is a constant called the *specific heat capacity,* which is measured in J/(kg-°C) in the MKS system.

REMEMBER

So it takes 4186 J of heat energy to warm up 1.0 kg of water 1.0°C. One calorie is defined as the amount of heat needed to heat 1.0 g of water 1.0°C, so 1 calorie equals 4.186 J. Nutritionists use the food energy term *Calorie* (capital C) to stand for 1000 calories, 1.0 kcal, so 1.0 Calorie equals 4186 J. And when you're speaking in terms of heat, you have another unit of measurement to deal with: the British Thermal Unit (Btu). 1.0 Btu is the amount of heat needed to raise one pound of water 1.0°F. To convert Btus to Joules, use the relation that 1 Btu equals 1055 J.

If you add heat to an object, raising its temperature from T_o to T_f, the amount of heat you need is expressed as:

$$\Delta Q = m \cdot c \cdot (T_f - T_o)$$

EXAMPLE

Q. You're heating a 1.0 kg copper block, specific heat capacity of 387 J/(kg-°C), raising its temperature by 45°C. What amount of heat do you have to apply?

A. The correct answer is 17,400 J.

1. **Use this equation:**

$$Q = m \cdot c \cdot \Delta T$$

2. **Plug in the numbers:**

$$Q = m \cdot c \cdot \Delta T = (387) \cdot (1.0) \cdot (45) = 17{,}400 \text{ J}$$

15. You're heating a 15.0 kg copper block, specific heat capacity of 387 J/(kg-°C), raising its temperature by 100°C. What heat do you have to apply?

Solve It

16. You're heating a 10.0 kg steel block, specific heat capacity of 562 J/(kg-°C), raising its temperature by 170°C. What heat do you have to apply?

Solve It

17. You're heating a 3.0 kg glass block, specific heat capacity of 840 J/(kg-°C), raising its temperature by 60°C. What heat do you have to apply?

Solve It

18. You're heating a 5.0 kg lead block, specific heat capacity of 128 J/(kg-°C), raising its temperature by 19°C. What heat do you have to apply?

Solve It

19. You're cooling a 10.0 kg lead block, specific heat capacity of 128 J/(kg-°C), lowering its temperature by 60°C. What heat do you have to extract?

Solve It

20. You're cooling a 80.0 kg glass block, specific heat capacity of 840 J/(kg-°C), lowering its temperature by 16°C. What heat do you have to extract?

Solve It

21. You put 7600 J into a 14 kg block of silver, specific heat capacity of 235 J/(kg-°C). How much have you raised its temperature?

Solve It

22. You add 10,000 J into a 8.0 kg block of copper, specific heat capacity of 387 J/(kg-°C). How much have you raised its temperature?

Solve It

Changes of Phase: Latent Heat

Heating blocks of lead is fine, but if you heat that lead enough, sooner or later it's going to melt. When it melts, its temperature stays the same until it liquefies, and then the temperature of the lead increases again as you add heat. So why does its temperature stay constant as it melts? Because the heat you applied went into melting the lead. There's a latent heat of melting that means that so many Joules must be applied per kilogram to make lead change phase from solid to liquid.

REMEMBER

The units of latent heat are J/kg.

There are three phase changes that matter can go through — solid, liquid, and gas — and each transition has a latent heat:

- **Solid to liquid:** The latent heat of melting (or heat of fusion), L_f, is the heat per kilogram needed to make the change between the solid and liquid phases (such as when water turns to ice).
- **Liquid to gas:** The latent heat of vaporization, L_v, is the heat per kilogram needed to make the change between the liquid and gas stages (such as when water boils).
- **Solid to gas:** The latent heat of sublimation, L_s, is the heat per kilogram needed to make the change between the solid and gas phases (such as the direct sublimation of dry ice (CO^2) to the vapor state).

The latent heat of fusion of water is about 3.35×10^5 J/kg. That means it takes 3.35×10^5 J of energy to melt 1 kg of ice.

Q. You have a glass of 50.0 g of water at room temperature, 25°C, but you'd prefer ice water at 0°C. How much ice at 0.0°C do you need to add?

A. The correct answer is 15.6 g.

1. The heat absorbed by the melting ice must equal the heat lost by the water you want to cool. Here's the heat lost by the water you're cooling:

$$\Delta Q_{water} = m \cdot c\ \Delta T = m \cdot c \cdot (T_f - T_o)$$

2. Plug in the numbers:

$$\Delta Q_{water} = m \cdot c \cdot \Delta T = m \cdot c \cdot (T_f - T_o) = (0.050)\ (4186)\ (0 - 25) = -5.23 \times 10^3 \text{ J}$$

3. So the water needs to lose 5.23×10^3 J. How much ice would that melt? That looks like this, where L_m is the latent heat of melting:

$$\Delta Q_{ice} = m_{ice} \cdot L_m$$

4. You know that for water, L_m is 3.35×10^5 J/kg, so you get this:

$$\Delta Q = m_{ice} \cdot L_m = m_{ice} \cdot 3.35 \triangle 10^5$$

5. You know that equation has to be equal to the heat lost by the water, so you can set it to:

$$\Delta Q_{ice} = \Delta Q_{water}$$

In other words:

$$m_{ice} = \frac{\Delta Q_{water}}{L_m} = \frac{5.23 \times 10^3 \text{ J}}{3.35 \times 10^5 \text{ J/kg}}$$

6. You know that the latent heat of melting for water is $L = 3.35 \text{ x } 10^5$ J/kg, which means that:

$$m_{ice} = \frac{5.23 \times 10^3}{3.35 \times 10^5} = 1.56 \times 10^{-2} \text{ kg}$$

So you need 1.56×10^{-2} kg, or 15.6 g of ice.

23. You have 100.0 g of coffee in your mug at 80°C. How much ice at 0.0°C would it take to cool 100.0 g of coffee at 80°C to 65°C?

Solve It

24. You have 200.0 g of cocoa at 90°C. How much ice at 0.0°C do you have to add to the cocoa (assuming that it has the specific heat capacity of water) to cool it down to 60°C?

Solve It

Answers to Problems about Thermodynamics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **–5° C**

1. Use this equation:

$$C = \frac{5}{9}(F - 32)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32) = (0.55) \cdot (23 - 32) = -5°C$$

2 **31.7° C**

1. Use this equation:

$$C = \frac{5}{9}(F - 32)$$

2. Plug in the numbers:

$$C = (F - 32) \cdot \frac{5}{9} = (0.55) \cdot (89 - 32) = 31.7°C$$

3 **64° F**

1. Use this equation:

$$F = \frac{9}{5}C + 32$$

2. Plug in the numbers:

$$F = \frac{9}{5}C + 32 = (1.8) \cdot (18) + 32 = 64°F$$

4 **291° K**

1. Use this equation:

$$K = C + 273.15$$

2. Plug in the numbers:

$$K = C + 273.15 = 18 + 273.15 = 291°\ K$$

5 **–255° C**

1. Use this equation:

$$C = K - 273.15$$

2. Plug in the numbers:

$$C = K - 273.15 = 18 - 273.15 = -255°\ C$$

6 **–357° F**

1. Convert Kelvins to Celsius:

$$C = K - 273 = 57 - 273 = -216°C$$

2. Convert Celsius to Fahrenheit:

$$F = C \cdot \frac{9}{5} + 32 = 357°F$$

7 **$1.0 + 2.3 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha\ L_o\ \triangle T = (2.3 \times 10^{-5}) \cdot (1.0) \cdot (100) = 2.3 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 2.3 \times 10^{-3}\ \text{m}$$

8 **$1.0 + 5.6 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T = (1.4 \times 10^{-5}) \cdot (2.0) \cdot (200) = 5.6 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 5.6 - 10^{-3}\ \text{m}$$

9 **$1.0 + 7.6 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T = (1.7 \times 10^{-5}) \cdot (1.5) \cdot (300) = 7.6 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 7.6 \times 10^{-3}\ \text{m}$$

10 **$1.0 + 2.9 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T = (2.9 \times 10^{-5}) \cdot (2.5) \cdot (40) = 2.9 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 2.9 \times 10^{-3}\ \text{m}$$

11 **$2.0 + 4.1 \times 10^{-3}\ \text{m}^3$**

1. Use this equation:

$$\triangle V = \beta\ V_o\ \triangle T$$

2. Plug in the numbers:

$$\triangle V = \beta \cdot V_o \cdot \triangle T = (6.9 \times 10^{-5}) \cdot (2.0) \cdot (30) = 4.1 \times 10^{-3}\ \text{m}^3$$

3. The final length is

$$V_f = V_o + \triangle V = 2.0 + 4.1 \times 10^{-3}\ \text{m}^3$$

12 **$2.0 + 2.0 \times 10^{-3}$ m^3**

1. Use this equation:

$$\triangle V = \beta \cdot V_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle V = \beta \cdot V_o \cdot \triangle T = (5.1 \times 10^{-5}) \cdot (2.0) \cdot (20) = 2.0 \times 10^{-3} \text{ m}^3$$

3. The final length is

$$V_f = V_o + \triangle V = 2.0 + 2.0 \times 10^{-3} \text{ m}^3$$

13 **$1.0 + 2.7 \times 10^{-4}$ m^3**

1. Use this equation:

$$\triangle V = \beta \cdot V_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle V = \beta \cdot V_o \cdot \triangle T = (1.0 \times 10^{-5}) \cdot (1.0) \cdot (27) = 2.7 \times 10^{-4} \text{ m}^3$$

3. The final length is

$$V_f = V_o + \triangle V = 1.0 + 2.7 \times 10^{-4} \text{ m}^3$$

14 **$3.0 + 2.3 \times 10^{-3}$ m^3**

1. Use this equation:

$$\triangle V = \beta \cdot V_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle V = \beta \cdot V_o \cdot \triangle T = (4.2 \times 10^{-5}) \cdot (3.0) \cdot (18) = 2.3 \times 10^{-3} \text{ m}^3$$

3. The final length is

$$V_f = V_o + \triangle V = 1.0 + 2.3 \times 10^{-3} \text{ m}^3$$

15 **5.8×10^5 J**

1. Use this equation:

$$Q = m \cdot c \ \triangle T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \triangle T = (387)\ (15.0)\ (100) = 5.8 \times 10^5 \text{ J}$$

16 **9.6×10^5 J**

1. Use this equation:

$$Q = m \cdot c \cdot \triangle T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \triangle T = (562)\ (10.0)\ (170) = 9.6 \times 10^5 \text{ J}$$

17 **1.5×10^5 J**

1. Use this equation:

$$Q = m \cdot c \cdot \triangle T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \triangle T = (840)\ (3.0)\ (60) = 1.5 \times 10^5 \text{ J}$$

18 **1.2×10^4 J**

1. Use this equation:

$$Q = m \cdot c \cdot \triangle T$$

2. Plug in the numbers:

$$Q = m\ c\ \triangle T = (128)\ (5.0)\ (19) = 1.2 \times 10^4 \text{ J}$$

19 **–7680 J**

1. Use this equation:

$$Q = m \cdot c \cdot \triangle T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \triangle T = (128)\ (10.0)\ (-60) = -7680$$

20 **-1.1×10^6 J**

1. Use this equation:

$$Q = m \cdot c \cdot \triangle T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \triangle T = (840)\ (80.0)\ (-16) = -1.1 \times 10^6 \text{ J}$$

21 **2.3° C**

1. Use this equation:

$$Q = m \cdot c \cdot \triangle T$$

2. Solve for $\triangle T$:

$$\frac{Q}{m \cdot c} = \Delta T$$

3. Plug in the numbers:

$$\frac{Q}{m \cdot c} = \Delta T = 7600/\left[(235)(14)\right] = 2.3^\circ\text{C}$$

22 **3.2° C**

1. Use this equation:

$$Q = m \cdot c\ \triangle T$$

2. Solve for $\triangle T$:

$$\frac{Q}{m \cdot c} = \Delta T$$

3. Plug in the numbers:

$$\frac{Q}{m \cdot c} = \Delta T = 10{,}000/\left[(387) \cdot (8.0)\right] = 3.2^\circ\text{C}$$

23 **10.4 g**

1. Calculate how much heat has to be lost by the coffee. Assuming it has the same specific heat capacity as water, that's:

$$\triangle Q_{cocoa} = m \cdot c \cdot \triangle T = m \cdot c \cdot (T_f - T_o)$$

2. Plug in the numbers:

$$\triangle Q_{cocoa} = m \cdot c \cdot \triangle T = m \cdot c \cdot (T_f - T_o) = (4186)\ (0.10)\ (80 - 65) = 6.28 \times 10^3 \text{ J}$$

3. How much ice do you need to remove 6.28×10^3 J? In this case, the heat supplied to the ice not only must melt the ice:

$$\triangle Q_{ice} = m_{ice} \cdot L_m$$

But this heat also needs to raise the temperature of the water that comes from melting the ice from 0°C to 65°C, so you have to add this:

$$\triangle Q_{ice} = m_{ice} \cdot L_m + m_{ice} \cdot c \cdot \triangle T = m_{ice} \cdot L_m + m_{ice} \cdot c \cdot (T_f - T_o)$$

4. This has to be equal to the heat lost by the coffee, so you get

$$6.28 \times 10^3 \text{ J} = m_{ice} \cdot [L_m + c \cdot (T_f - T_o)]$$

5. Solve for m_{ice}:

$$m_{ice} = \frac{6.28 \times 10^3 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]}$$

6. Plug in the numbers:

$$m_{ice} = \frac{6.28 \times 10^3 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]} = \frac{6.28 \times 10^3 \text{ J}}{\left[3.35 \times 10^5 + 4186 \cdot (65 - 0)\right]} = 0.0104 \text{ kg}$$

24 **42 g**

1. Find how much heat has to be lost by the cocoa. Assuming it has the same specific heat capacity as water, that's:

$$\triangle Q_{cocoa} = m \cdot c \cdot \triangle T = cm(T_f - T_o)$$

2. Plug in the numbers:

$$\triangle Q_{cocoa} = m \cdot c \cdot \triangle T = m \cdot c \cdot (T_f - T_o) = (0.20)\ (4186)\ (90 - 60) = 2.5 \times 10^4 \text{ J}$$

3. How much ice do you need to supply 2.5×10^4 J? In this case, the heat supplied to the ice, not only must melt the ice:

$$\triangle Q_{ice} = m_{ice} \cdot L_m$$

But this heat also needs to raise the temperature of the water that comes from melting the ice from 0°C to 60°C, so you have to add this:

$$\triangle Q_{ice} = m_{ice} \cdot L_m + m_{ice} \cdot c\ \triangle T = m_{ice} \cdot L_m + m_{ice} \cdot c \cdot (T_f - T_o)$$

4. This has to be equal to the heat lost by the cocoa, so you get

$$2.5 \times 10^4 \text{ J} = m_{ice} \cdot [L_m + c \cdot (T_f - T_o)]$$

5. Solve for m_{ice}:

$$m_{ice} = \frac{2.5 \times 10^4 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]}$$

6. Plug in the numbers:

$$m_{ice} = \frac{2.5 \times 10^4 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]} = \frac{2.5 \times 10^4 \text{ J}}{\left[3.35 \times 10^5 + 4186 \cdot (60 - 0)\right]} = 0.042 \text{ kg}$$

15. You have 29.0 g of zinc, atomic mass 65.41 u. How many atoms do you have?

Solve It

16. You have 3.0 g of copper, atomic mass 63.546 u. How many atoms do you have?

Solve It

Ideally Speaking: The Ideal Gas Law

You can relate the pressure, volume, and temperature of an ideal gas with the ideal gas equation. An *ideal gas* is one whose molecules act like points; no interaction occurs among molecules except *elastic collisions* (that is, where kinetic energy is conserved). In practice, all gases act like ideal gases to some extent, so the ideal gas law holds fairly well. Here's that law:

$$P \cdot V = n \cdot R \cdot T$$

Here, P is pressure; n is the number of moles of gas you have; R is the *universal gas constant,* which has a value of 8.31 J(/mole-K); and T is measured in Kelvins. The volume, V, is measured in cubic meters, m^3. Sometimes, you'll see volume given in liters, where 1.0 L = 10^{-3} m^3. Using this law, you can predict the pressure of an ideal gas, given how much you have of it, its temperature, and the volume you've enclosed it in.

One mole of ideal gas takes up 22.4 L of volume at 0°C and one atmosphere pressure, which is 1.013×10^5 N/m^2, where N/m^2 (Newtons per square meter) is given its own units, Pascals, abbreviated Pa.

You can also write the ideal gas law a little differently by using Avogadro's Number, N_A, and the total number of molecules, N:

$$P \cdot V = n \cdot R \cdot T = (N/N_A)\, R \cdot T$$

The constant R/N_A is also called Boltzmann's constant, k, and it has a value of 1.38×10^{-23} J/K. Using this constant, the ideal gas law becomes

$$P \cdot V = N \cdot k \cdot T$$

EXAMPLE

Q. You have 1.0 moles of air in your tire at 0°C, volume 10.0 L. What is the gauge pressure within the tire?

A. The correct answer is 1.3×10^5 Pa.

1. Use this equation:

$$PV = nRT$$

2. Solve for P:

$$P = nRT / V$$

3. Plug in the numbers to get the pressure pushing out:

$$P = nRT / V = (1.0)\ (8.31)\ (273) / (10 \times 10^{-3}) = 2.3 \times 10^5 \text{ Pa}$$

4. Subtract the pressure from the surrounding air, which pushes in, assuming that the air is at 0°C too:

$$P = 2.3 \times 10^5 - 1.013 \times 10^5 = 1.3 \times 10^5 \text{ Pa}$$

17. You have 2.3 moles of air in your tire at 0°C, volume 12.0 L. What pressure is the tire inflated to?

Solve It

18. You have a bottle of 2.0 moles of gas, volume 1.0 L, temperature 100°C. What is the pressure inside the bottle?

Solve It